Quasi-Failure Analysis on Resonant Demolition of Random Structural Systems

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Introduction

PARAMETER uncertainty of structural systems is inherent in most engineering problems. Reliability analysis can help the designer to establish acceptable tolerances on structures and to determine the fluctuations of the system parameters for safe operations. Up to now, failure problems of uncertain vibration structure systems have followed two paths. One is failure research on the basis of the responses (displacement, stress, etc.) of forced vibration.[1–8] The other is failure research on the basis of the relation between natural frequency and forcing frequency of vibration systems at resonance and nonresonance.[9] In this Note, the reliability analysis method of avoiding resonance for multi-degree-of-freedom structural systems is presented. The failure model and the failure probability of the random systems are defined. When the stochastic perturbation method and the reliability theory are used, the formulas of quasi-failure probability for resonant problems of random structures are obtained.

Random Natural Frequency Problem

The generalized problem is to consider the probabilistic effects of natural frequencies and its natural modes through random parameter vector $\boldsymbol{b} = (b_i)_s$. This can include the probabilistic distributions of all discretized random characteristics. For reference purposes, the general case considered is the following eigenvalue problem:

$$\left(\boldsymbol{K} - \omega_k^2 \boldsymbol{M}\right)\boldsymbol{X}_k = 0 \tag{1}$$

$$\boldsymbol{X}_k^T \boldsymbol{M} \boldsymbol{X}_k = 1 \tag{2}$$

where ω_k and $\boldsymbol{X}_k$ are the kth natural frequency and its associated natural mode. In common structural applications, the $n \times n$ matrices $\boldsymbol{K}$ and $\boldsymbol{M}$ are, respectively, the stiffness and mass matrices of the structure.

The $\boldsymbol{K}$ and $\boldsymbol{M}$ and ω_k and $\boldsymbol{X}_k$ are expressed about the mean value part and random perturbation part

$$\boldsymbol{K} \approx \boldsymbol{K}_0 + \Delta \boldsymbol{K}, \qquad \boldsymbol{M} \approx \boldsymbol{M}_0 + \Delta \boldsymbol{M} \tag{3}$$

$$\omega_k \approx \omega_{k0} + \Delta\omega_k, \qquad \boldsymbol{X}_k \approx \boldsymbol{X}_{k0} + \Delta \boldsymbol{X}_k \tag{4}$$

When Eqs. (3) and (4) are substituted into Eqs. (1) and (2), the first perturbation is obtained:

$$\left(\boldsymbol{K}_0 - \omega_{k0}^2 \boldsymbol{M}_0\right)\boldsymbol{X}_{k0} = 0 \tag{5}$$

$$\boldsymbol{X}_{k0}^T \boldsymbol{M}_0 \boldsymbol{X}_{k0} = 1 \tag{6}$$

$$\Delta\omega_k = (1/2\omega_{k0})\left[\boldsymbol{X}_{k0}^T\left(\Delta\boldsymbol{K} - \omega_{k0}^2 \Delta\boldsymbol{M}\right)\boldsymbol{X}_{k0}\right] \tag{7}$$

$$\boldsymbol{X}_{k0}^T \boldsymbol{M}_0 \Delta\boldsymbol{X}_{k0} = -\tfrac{1}{2}\boldsymbol{X}_{k0}^T \Delta\boldsymbol{M}\boldsymbol{X}_{k0} \tag{8}$$

where ω_{k0} and $\boldsymbol{X}_{k0}$ are the kth natural frequency and its associated natural mode of mean value system.

$\Delta\boldsymbol{K}$, $\Delta\boldsymbol{M}$, and $\Delta\omega_k$ are expanded about an $E(\boldsymbol{b}) = \bar{\boldsymbol{b}}$ first-order Taylor series, and when they are substituted into Eq. (7), the variance and covariance of the natural frequency is obtained:

$$\mathrm{cov}(\omega_k, \omega_l) = \sum_{i=1}^{q}\sum_{j=1}^{q}\left(\frac{\partial\bar{\omega}_k}{\partial b_i}\right)\left(\frac{\partial\bar{\omega}_l}{\partial b_j}\right)\mathrm{cov}(b_i, b_j) \tag{9}$$

where

$$\frac{\partial\bar{\omega}_k}{\partial b_i} = \frac{1}{2\omega_{k0}}\left[\boldsymbol{X}_{k0}^T\left(\frac{\partial\bar{\boldsymbol{K}}}{\partial b_i} - \omega_{k0}^2\frac{\partial\bar{\boldsymbol{M}}}{\partial b_i}\right)\boldsymbol{X}_{k0}\right] \tag{10}$$

where $\partial\bar{A}/\partial b_i = \partial A(\bar{\boldsymbol{b}})/\partial b_i$ is the value of A evaluated at $\bar{\boldsymbol{b}}$.

Quasi-Failure Analysis

A great many of responses do not exceed the thresholds; however, the system may encounter resonance, which can cause the failure of structure systems, or the state of structure systems is in what may be called the quasi-failure state. In this way, the structure system is guaranteed from avoiding resonance with appropriate probability. According to the reliability theory, the state function of random systems is defined as

$$g_{ij}(p_j, \omega_i) = |p_j - \omega_i|, \qquad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m \tag{11}$$

where p_j is the jth forcing frequency. According to the relation criterion of natural frequency ω_i and forcing frequency p_j, the quasi-failure state of structure systems is represented as

$$g_{ij}(p_j, \omega_i) = |p_j - \omega_i| \le \gamma \tag{12}$$

where γ is a specified range. The mean value and variance state functions g_{ij} are

$$\mu_{g_{ij}} = E(g_{ij}) = E|p_j - \omega_i| \tag{13}$$

$$\sigma_{g_{ij}}^2 = \mathrm{var}(g_{ij}) = \sigma_{p_j}^2 + \sigma_{\omega_i}^2 \tag{14}$$

The probability of the system in which resonance occurs, namely, probability of the quasi-failure state, is

$$P_f^{ij} = P^{ij}(-\gamma \le p_j - \omega_i \le \gamma) \tag{15}$$

In structure systems, resonance has occurred because of any forcing frequency that is in the vicinity of any natural frequency. The whole systems are considered to be in quasi-failure states. Thus, the structure system, to which forcing frequencies and natural frequencies are applied to analyze quasi-failure state, is a series system. The probability of the quasi-failure state of the whole system is represented as

$$P_f = 1 - \prod_{i=1}^{n}\prod_{j=1}^{m}\left(1 - P_f^{ij}\right) \tag{16}$$

Numerical Example

The example is described as follows. A vehicle runs on a plane road with velocity v (Fig. 1). When it undergoes the rough road, vibration must occur. The shape of the protruding part is signified as $y = h\sin pt$, where $p = \pi v/l$ and l is length of protruding part. The random parameters m_1, m_2, k_1, k_2, and l are mutual independent random variables. The statistic quantities of the random parameters are $(\mu_{m_1}, \sigma_{m_1}) = (4500, 225)$ kg, $(\mu_{m_2}, \sigma_{m_2}) = (45{,}000, 2250)$ kg, $(\mu_{k_1}, \sigma_{k_1}) = (1.683 \times 10^7, 8.415 \times 10^5)$ N/m, $(\mu_{k_2}, \sigma_{k_2}) = (3.136 \times 10^6, 1.568 \times 10^5)$ N/m,

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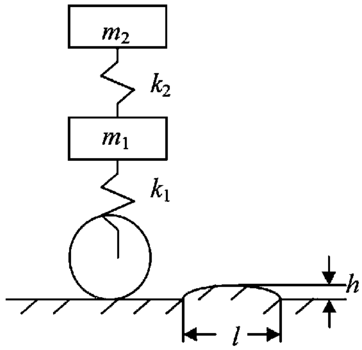

Fig. 1 System model.

$(\mu_l, \sigma_l) = (3, 0.15)$ m, respectively, and $v = 20$ m/s. From experience, the specified range of γ should be chosen between 10 and 15% of the first natural frequency.

The differential equation of the vehicle system is

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = k_1 h \sin pt$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0$$

The probability of the quasi-failure state of the whole system is

$$P_f = 0.0$$

Thus, resonant vibration cannot occur in the system.

It is obvious that vehicle runs on the rough road avoiding velocities of 7.309 m/s or 63.694 m/s. Here the probability of the quasi-failure state of whole system is

$$P_f = 0.9774$$

Thus, resonant vibration can occur in the system.

Conclusions

Frequency analysis is important for reliability problems of dynamically uncertain structures. This Note presents a frequency reliability analysis method. According to the criterion of the frequency relation, the structural system with resonant failure is defined as a series system. One of the reliability problems of random structural systems is preferably solved using frequency analysis and reliability theory. Numerical results are presented.

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Topology Optimization for Maximum Natural Frequency Using Simulated Annealing and Morphological Representation

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I. Introduction

FOR topology optimization of continuum structures, the most common strategy is based on discretizing the allowable space into finite elements (FEs) and considering the material density or amount of material within each element as design variables. When the relevant optimization problem is solved, the optimum geometry thus emerges from the resulting optimum distribution/arrangement of material within the design space. The homogenization[1] and material density[2] methods are the popular methods based on such a strategy, and many static and dynamic topology/shape optimization problems have been solved using them.[3–5] However, because this strategy uses continuous design variables, the resulting elements can be of intermediate densities (gray areas) spanning the range from completely empty to completely filled with material. Hence, the final geometry has to be interpreted from these results, usually by imposing some threshold density value, which may be arbitrary and thus lead to uncertainties. Furthermore, with no controls or constraints over the arrangement of material, the optimum result may contain checkerboard patterns and floating elements disconnected from the main structural body.

To overcome these drawbacks, the recently developed morphological representation scheme[6] is used to define structural geometry in this work, optimizing structures to achieve maximum natural frequency with a constraint on the volume of material. Because changes in topology are discrete changes, the problem is treated as a discrete optimization using the simulated annealing (SA) algorithm inasmuch as the SA is readily adaptable to various classes of problems and has the ability to escape from local optima with the possibility of reaching the global optimum. The SA also does not enumerate

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